# Single quadrotor dynamics

## General

The quad-rotor has 4 body arms and 4 electrical motors – each produces thrust dependent on the rotation speed of the rotors blades. At nominal state, all the rotors are rotating at the same constant speed, with direction shown in Fig.1. The engines are numbered 1-4 as shown in the figure.



Figure 1

Rotors 1 and 3 are both Clockwise, rotors 2 and 4 are both CounterClockwise.

## Analysis assumptions

1. **Ground effect is *not* taken in account** because the desired balance point for investigation is above ground. (usually G.E is considered when at least away)

1.a disturbance on the payload, as result of **downwash** is also not taken in account. Assuming the payload cable (or rod) is long enough.

1. The quad body and parts are assumed to be **rigid**. *No* elasticity is considered.
2. Body geometry and mass is (almost) **symmetrical**. Hence the Inertia matrix is pure diagonal. (this issue might need to be re-considered in later phase of the work)

## The quadrotor parts and elements

The quad is assembled of few mechanical and electrical parts:

1. Propeller (x4) : 2 are appropriate for CW direction rotation, and 2 for CCW rotation
2. Electric motor (x4)
3. ESC (electronic speed control) – controls the electric motors torque output, by input of a varying voltage.
4. MicroController : the electric unit that controls all the ESCs.
5. Battery (usually LiPo type, with several other varying properties i.e : maximum possible current, nominal voltage)
6. IMU sensor (6 to 10 D.O.F) : measures the unit linear accelerations, and angular velocities. Might also measure barometric pressure. Might also include a Magnetometer.

Each of these parts have a certain behavior. Their output will be in accordance with some input.

The elements equations are listed bellow, and the more elaborated models are described in the Appendix XX.

## motion equations per the Newton-Euler method

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## motion equations per the Lagrange method

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1. **Forces** acting on the rotors and the body frame :

The External forces are derived from aerodynamic reactions - creating forces and moments on the quad body(frame) and the motors themselves and hence the body hub. by a combination of momentum and blade element theory (as in helicopters).

All the next mentioned forces and moments are in BODY framework.

1. Thrust of *each* rotor (i = 1..4) , acts perpendicular to the blades plane:
2. Side force on each of the motors’ Hubs. it is the outcome of the forces on the blades in their horizontal direction.

The resultant **moments** on the system are:

1. Drag moment - the result of moements by drag forces on the blades ( in their horizontal direction )
2. Rolling moment - integration over the entire rotor of the lift of each section acting at a given radius

2. **Kinematics** , and rotation matrices

I will consider the general coordinates of the quad:

is the location in space. Relative to inertial coor. system.

is the orientation relative to the inertial coor. system.

Hence the 6 D-O-F.

Description of the rotaion between vector in the body coor.system to description in the inretial frame-work:

I will use the ‘Tait–Bryan angles’ notion.

rotation about axis in Clockwise direction (negative right hand direction)

rotation about Y’ axis in Clockwise direction (negative right hand direction)

(Y’ is the intermidiate Y-axis of the ‘temporary’ coordinate system of euler angles)

rotation about axis in Clockwise direction (negative right hand direction)

and so the overall rotation matrix (by matrix multiplication) from Body to Inertial is :

( the other-way rotation matrix will be )

When given Body coordinates one can represent them in Inertial system by:

Angular velocity vector of the quad is :

For describing that vector in Body-coor.system we need to describe it in the relevant axes :

=

=

Define =

Using the 1st vector definition above we get , in body coordinates system , the (relative) angular rate :

= =

another thing the is influenced by the rotated coor.system is Moment of Inertia.

The matrix I = is in Body system.

The inertia in inertial system is : (rotated about the axis of rotation).

3. Energies of the system :

In order to derive the equations of motion, I’ll use the Lagraunge method.

Therefor I’ll use the lagrangian which is :

*Kinetic* energy ( translational + rotational contributioins ) :

*Potential* energy:

(z positive is downwards)

Position of vector in Body system is : location of the C.G with addition of location in that system:

* V=

As long as the system is only a rigid body then v= , and (the angular rates we showed above).

T=

(again – it would be more complicated argument if we considered a non symetric rigid body)

The Lagrange equation are:

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Some of the used trigonometric identities are:

the general forces are :

– drag of the body frame and

- drag on the body frame parts, that are below the rotors and affected by the rotors downwash.

# Appendix A – system elements separate models

## Propeller

Thrust is related to rotational speed.

## ESC

## Electric motor